Problem 16

Time standards are now based on atomic clocks. A promising second standard is based on *pulsars*, which are rotating neutron stars (highly compact stars consisting only of neutrons). Some rotate at a rate that is highly stable, sending out a radio beacon that sweeps briefly across Earth once with each rotation, like a lighthouse beacon. Pulsar PSR $1937 + 21$ is an example; it rotates once every $1.557\,806\,448\,872\,75 \pm 3$ ms, where the trailing ± 3 indicates the uncertainty in the last decimal place (it does not mean ± 3 ms). (a) How many rotations does PSR $1937 + 21$ make in 7.00 days? (b) How much time does the pulsar take to rotate exactly one million times and (c) what is the associated uncertainty?

Solution

Determine the number of rotations by using conversion factors, beginning with 7.00 days.

$$7.00 \text{ days} \times \frac{24 \text{ hours}}{1 \text{ day}} \times \frac{60 \text{ min}}{1 \text{ hour}} \times \frac{60 \text{ s}}{1 \text{ min}} \times \frac{10^3 \text{ ms}}{1 \text{ s}} \times \frac{1 \text{ rotation}}{1.557\,806 \text{ ms}} \approx 3.88 \times 10^8 \text{ rotations}$$

If PSR $1937 + 21$ rotates once every $1.557\,806\,448\,872\,75 \pm 3$ ms, then it rotates a million times every

$$10^6 \times 1.557\,806\,448\,872\,75 \text{ ms} \times \frac{1 \text{ s}}{10^3 \text{ ms}}$$

$$1557.806\,448\,872\,75 \text{ s},$$

and the uncertainty associated with the 5 at the end is $\pm 3 \times 10^{-11}$. What this means is there are upper and lower bounds for the time; the true value lies somewhere between them.

$$\text{Upper Bound}: \quad 1557.806\,448\,872\,78 \text{ s}$$

$$\text{Lower Bound}: \quad 1557.806\,448\,872\,72 \text{ s}$$